Power Output Of Segmented MHD Faraday Generator With Voltage Consolidation

Author(s): D. R. Trung, S. W. Simpson, and H. Messerle

Session Name: Electrical Components

SEAM: 19 (1981)

SEAM EDX URL: https://edx.netl.doe.gov/dataset/seam-19

EDX Paper ID: 916

POWER OUTPUT OF SEGMENTED MHD FARADAY GENERATOR WITH VOLTAGE CONSOLIDATION

D.T. Trung, S.W. Simpson and H.K. Messerle
School of Electrical Engineering,
University of Sydney
N.S.W. 2006, Australia

Abstract

A two-dimensional analytical model of an MHD duct with finite electrodes is derived. The model, which is efficient in terms of computer time, is used to study some aspects of voltage consolidation circuits for Faraday generators.

I. Introduction

Rosa[1] and Lowenstein[2] have proposed various circuits for consolidation and control of the power output of MHD generators. These circuits are equivalent to the one shown in Fig. 1: p pairs of electrodes are interconnected via Hall compensation voltage sources and the output is fed to a single inverter. Power either flows from or to the various Hall compensation circuits depending on the location of the inverter connections. For the connections shown in Fig. 1, power is supplied to all the circuits to enable current to flow to the inverter against the Hall potential gradient. This power is derived by rectifying a portion of the inverter output, so that a part of the ac power is recirculated through the circuits defined by the $V_{cA,j}$ and $V_{ck,j}$ compensation voltages (circuit "A" of Ref. 1). If the inverter connections are located in the centre of the group, half of the $V_{cA,j}$ and $V_{ck,j}$ supply power and the other half absorb power. It is then possible to interconnect the compensation circuits so that no nett power comes from the inverter output (circuit "B" of Ref. 1; configuration 1 of Ref.2). Finally, making connections to the opposite ends of the group means that all the compensation circuits supply power to the inverter (circuit "F" of Ref. 1). In this work, the actual circuit implementation is not considered; calculations of the power output, recirculated power, etc. are made for the idealised circuit of Fig. 1.

Initially the performance of Rosa's consolidation circuit "A" was analysed by coupling it with a simple equivalent circuit model for each electrode pair (Fig. 2). In Fig. 2, V_{hx} is the Hall voltage developed along the channel by the current I_y flowing between the electrodes; V_{hy} is the Hall voltage produced by the current I_x flowing along the channel; R_x and R_y are average resistances of the duct, given by $R_x = s/\sigma hw$ and $R_y = h/\sigma sw$ (h, channel height, w, channel width, s, electrode pitch); and $U_g = uBh$ is the voltage developed by fluid velocity u in the duct. The combined circuit was solved using a general circuit analysis computer program. It was found that the recirculated power could constitute a large fraction (20% - 40%) of the total inverter output.[3]

The equivalent circuit simulation of the MHD flow described above offers only an approximate solution. In order to account more accurately for the interaction between electrodes in the duct, a solution to Laplace's Equation for the current stream function (or potential etc.) is required.[4] This equation is often solved numerically using a grid of points in the plane normal to $\underset{\sim}{B}$ in the duct. However, for the present application, where we wish to find the combined V-I characteristics of a group of electrodes, such an approach would require a large amount of computer time. Instead we have used an analytical method which is more convenient and more efficient in terms of computer time.

II. Mathematical Model

Following Solbes and Lowenstein[5], we consider the two dimensional current distribution in a linear generator with constant height, uniform velocity and uniform electrical properties (Fig. 3).

The complex current potential, Z, is defined by

$$\frac{dZ}{dz} = J_x - iJ_y,$$

where $z = x + iy$ and $i = \sqrt{-1}$.

For a finite length Faraday generator, Z is related to J_A and J_k, the current densities at the anode and cathode walls, as follows:[5]

$$Z = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{J_A(x')\cos[k(z-x'+ih/2)]}{2\pi k \sinh(kh)}dx'dk - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{J_k(x')\cos[k(z-x'-ih/2)]}{2\pi k \sinh(kh)}dx'dk \quad (1)$$

where $J_A(x) = J_y(x, h/2)$ and $J_k(x) = J_y(x,-h/2)$.

The electric potential θ is related to Z by

$$\phi = \text{Re}\,(iuB_z z - Z\frac{(1+i\beta)}{\sigma}) \quad (2)$$

where σ is the conductivity and β is the Hall parameter.

In the real situation, $J_A(x)$ and $J_k(x)$ are not known : the electrodes are specified as regions of constant potential. Here the problem is simplified by assuming a uniform current flow over each electrode, and the electrode width used in calculations is reduced to account for current concentration due to the Hall effect. We consider a uniform arrangement of electrodes of width ℓ and spacing s. Current $-I_{A,n}$ enters the duct uniformly over the area of the nth anode, and current $-I_{k,n}$ leaves the duct uniformly over the area of the nth cathode. Voltage differences relevant to the external circuits are:

(i) the voltage developed between cathode j and cathode j + 1,

$$\Delta V_{k,j} = \phi\,(s(j+1), -h/2) - \phi\,(sj, -h/2),$$

(ii) the voltage developed between anode j and anode j + 1,

$$\Delta V_{A,j} = \phi\,(s(j+1), h/2) - \phi\,(sj, h/2),$$

and (iii) the voltage developed between anode j and cathode j,

$$\Delta V_j = \phi(sj, -h/2) - \phi(sj, h/2).$$

Relations (1) and (2) can be combined to give expressions for these voltages when there is only current flow $I_{k,o}$ and $I_{A,o}$ from the electrodes at x=0:

$$\Delta V_{k,j} = \frac{h}{\sigma \ell w}[C_1 I_{A,o} - (C_2 + \beta C_3) I_{k,o}]$$

$$\Delta V_{A,j} = \frac{h}{\sigma \ell w}[(C_2 - \beta C_3) I_{A,o} - C_1 I_{k,o}]$$

$$\Delta V_j = uB_z h + \frac{h}{\sigma \ell w}[(C_4 + \beta C_5) I_{A,o} + (C_4 - \beta C_5) I_{k,o}]$$

where

$$C_1 = \int_{-\infty}^{\infty} \frac{dk\ \{\cos(ksj) - \cos[ks(j+1)]\}\sin(k\ell/2)}{\pi h k^2 \sinh(kh)},$$

$$C_2 = \int_{-\infty}^{\infty} \frac{dk\ \{\cos(ksj) - \cos[ks(j+1)]\}\sin(k\ell/2)}{\pi h k^2 \tanh(kh)},$$

$$C_3 = \int_{-\infty}^{\infty} \frac{dk\ \{\sin[ks(j+1)] - \sin(ksj)\}\sin(k\ell/2)}{\pi h k^2},$$

$$C_4 = \int_{-\infty}^{\infty} \frac{dk\ [\cosh(kh)-1]\cos(ksj)\sin(k\ell/2)}{\pi h k^2 \sinh kh},$$

$$C_5 = \int_{-\infty}^{\infty} \frac{dk\ \sin(ksj)\sin(k\ell/2)}{hk^2},$$

and w is the channel width. $C_1 - C_5$ were evaluated by contour integration, giving:

$$C_1 = \delta f_1^{j+1} - \delta f_1^{j},\quad C_2 = \delta f_2^{j+1} - \delta f_2^{j},$$
$$C_3 = \delta f_3^{j+1} - \delta f_3^{j},\quad C_4 = \delta f_1^{j} - \delta f_2^{j}, \qquad (3)$$
$$C_5 = \delta f_3^{j},$$

where $\delta f_q^{\,k} = f_q\left(\frac{2sk+\ell}{2h}\right) - f_q\left(\frac{2sk-\ell}{2h}\right)$

for q = 1,2,3, and

$$f_1(x) = \frac{1}{\pi^2} d(1 + e^{-\pi x}) + \frac{x^2}{4} + \frac{1}{12},$$

$$f_2(x) = \frac{1}{\pi^2} d(1 - e^{-\pi x}) + \frac{x^2}{4} - \frac{1}{6},$$

$$f_3(x) = |x|/2,$$

with the dilogarithm function[6],

$$d(y) = -\int_1^y \frac{\ell n\ t}{t-1} dt.$$

Both Solbes and Lowenstein[5] and Kuo et al[7] have calculated the voltages produced by delta function current sources at the channel walls; however, their results differ. If we take the corresponding limit of the present result, we find agreement with the Solbes and Lowenstein expression.

Since a current distribution rather than a potential distribution has been specified at the electrode surfaces, the entire solution can be obtained by superposition. To find the voltage produced by electrode currents $I_{k,n}$ and $I_{A,n}$ at position j, j is replaced by j-n in Eqs. (3). The voltages when all electrode currents are present are then found by summing the contributions from each electrode current. The electrode currents can be arranged in a linear array I, and the voltages in an array V, as follows:

$$I^t = (I_{A,o}\ldots\ldots I_{A,m-1},\ I_{k,o}\ldots\ldots I_{k,m-1})$$

$$V^t = (\Delta V_o - uB_z h,\ \Delta V_{A,o}\ldots\Delta V_{A,m-2}, \Delta V_{k,o}\ldots\Delta V_{k,m-2}),$$

where 2m is the total number of electrodes. Then Eqs.(3) can be used to calculate the elements of a 2m-1 x 2m matrix A which relates I and V:

$$V = \frac{h}{\sigma \ell w} AI$$

To complete the solution, the Faraday generator requirement that

$$\sum_{n=o}^{m-1} (I_{k,n} - I_{A,n}) = 0 \qquad (4)$$

must be used. If I^1 is a vector of length 2m-1 generated by deleting any element of I, one can write $I = FI^1$ where F is a 2m x 2m-1 matrix which can readily be found from Eq. (4). It follows that

$$V = \frac{h}{\sigma \ell w} AFI^1,$$

where AF is a 2m-1 x 2m-1 "transresistance matrix", depending only on s, h, ℓ and β, which defines the behaviour of the duct in an external circuit. A polynomial approximation was used for computer evaluation of the function d(y) [Eqs.(3)]. For a duct with 2m electrodes, this polynomial is evaluated 2m times to determine all elements of AF; thus computing time is short.

III. Application to Voltage Consolidation Circuits

For the consolidation circuit under consideration (Fig.1), there are 2p electrode currents in each consolidated group, and initially all groups are taken to be identical. In this case the general procedure described in the previous section is followed except that, in calculating the contribution to a given voltage difference from a given electrode current, contributions from the same current in all groups must be summed. The duct is taken to be infinite in that contributions from distant groups are added until they become negligible. The result of the calculation is a 2p-1 x 2p-1 transresistance matrix for a consolidated group, relating the electrode current vector I^1 (with 2p-1 elements) to the voltage vector V.

The power output of a consolidated group can be written

$$P_{tot} = -V_c^t I_c + uB_z h I_o,$$

where the vectors V_c and I_c are defined by

$$I_c^t = (\ldots.I_{cA,j}\ldots.,\ldots.I_{ck,j}\ldots.,I_o)$$

and

$$V_c^t = (\ldots.V_{cA,j}\ldots.,\ldots.V_{ck,j}\ldots.,V_o - uB_z h),$$

and the $I_{cA,j}$ and $I_{ck,j}$ are the currents flowing in the Hall compensation circuits. I_c and V_c are linearly related to I^1 and V; using the transresistance matrix, matrices S_1 and S_2 can be found such that $I_c = S_1 I^1$ and $V_c = S_2 I^1$. Defining a normalised current vector by $R = -I^1/I_o$, we find

$$P_{tot} = -R^t S_1^t S_2 R I_o^2 + uB_z h I_o. \qquad (5)$$

The electrical efficiency η_e is given by

$$\eta_e = P_{tot}/uB_z hI_o. \qquad (6)$$

Eliminating I_o between Eqs.(5) and (6),

$$\frac{P_{tot}}{(uB_z h)^2} = \frac{\eta_e(1-\eta_e)}{R^t S_1^t S_2 R} \qquad (7)$$

This result shows the usual dependence of P_{tot} on η_e, as well as a term giving the dependence on the current arrangement in a consolidated group.

The term $R^t S_1 {}^t S_2 R$ in Eq.(7) is quadratic in the current ratios, $r_{A,j}=-I_{A,j}/I_o$ and $r_{k,j}=-I_{k,j}/I_o$, and can be minimised analytically by solving the set of equations

$$\frac{\partial}{\partial r_{A,j}} (R^t S_1^t S_2 R) = 0$$

$$\frac{\partial}{\partial r_{k,j}} (R^t S_1^t S_2 R) = 0$$

together with

$$\sum_{j=1}^{p} r_{A,j} = 1.$$

The outcome of this minimisation is that, for the uniform arrangement of electrodes considered, the optimum value of $P_{tot}/\eta_e(1-\eta_e)$ always occurs when the electrode currents are equal. This result reflects the fact that the external circuit here (Fig. 1) contains no dissipative elements, and a uniform current distribution minimises losses in the internal resistance of the duct.

In the uniform current case, the ratio F of the total power handling capacity of the consolidation elements to the power generated P_{tot} is given by

$$F = -p\theta\, \Delta V_x / V_y \qquad (8)$$

where ΔV_x is the Hall voltage between electrodes, V_y is the Faraday voltage, and θ depends on the circuit configuration as follows:

(i) For connections made at opposite ends of the group[1] (Fig. 1),

$$\theta = 1 - p^{-1}.$$

(ii) For connections made in the centre,[2]

$$\theta = \tfrac{1}{2} \qquad (p\ \text{even})$$

$$\theta = \tfrac{1}{2}(1 - p^{-2}) \qquad (p\ \text{odd}).$$

The effective Hall parameter in a duct with finite electrodes is given approximately by[8]

$$\beta_{eff} = \frac{\beta}{1 + (\beta-0.44)s/h}$$

Using this value to calculate ΔV_x in Eq.(8) and the relation $P_{tot}=-I_o V_y$ together with Eq.(6) to eliminate V_y, we find

$$\frac{ps}{h} = \frac{F}{\theta}\,\frac{\eta_e}{1-\eta_e}\,\frac{1+(\beta-0.44)s/h}{\beta} \qquad (9)$$

Equation (9) expresses the length of duct consolidated (normalised to the duct height) in terms of F and η_e. For $s/h \ll 1$, $\eta_e \approx 0.75$ and $\beta \approx 3$, $ps/h \approx F/\theta$, so that consolidation can only extend over a length equal to a small fraction of the channel height if the recirculated power is to remain a small fraction of the power output. Figure 4 shows the number of electrode pairs p which can be consolidated as a function of s/h for various values of F and η_e. For Fig. 4, β=3 and θ=0.5.

IV. Faults in Voltage Consolidation Circuits

The model described above can also be used to investigate situations in which one consolidated group of electrodes in a duct is affected by a fault. One of the simplest cases which can be considered is the failure of one electrode in a group to conduct a current. We have used the techniques described in the previous section to calculate the current arrangement in the remaining electrodes of the faulty group which gives maximum power output. In making the calculation the effect of this current arrangement on the power output of all groups was taken into account. The constraint imposed by requiring one current to be fixed at zero was handled in a similar way to the constraint of Eq. (4).

Table 1 shows the optimum current distribution and Table 2, the interelectrode voltage distribution, for a group of 7 electrode pairs with anode 5 faulty. Conditions for Tables 1 and 2 are $s/h = 0.04$, β=3, η_e=0.5 and ℓ/s=0.1; the currents and voltages are normalised to their values in a uniform distribution. The power loss in this situation is only 5.9% of the power output per electrode pair in the uniform case; however, Table 2 shows an increase of 20% in the voltage between anodes 6 and 7. Such an increase is undesirable from the point of view of interelectrode breakdown. Table 3 shows a simpler arrangement of electrode currents which leads to the voltage distribution given in Table 4. In this case the maximum interelectrode voltage is increased by 7% and the power loss is 16% of the power output per electrode pair.

V. Concluding Remarks

In this paper, we have applied an analytical model of MHD ducts to a very basic study of consolidation circuits for Faraday generators. In using the model, we have neglected boundary layer phenomena involving velocity and conductivity gradients, arc spot formation, wall conduction, and current concentration on electrode edges due to the Hall effect. Varying current density across an electrode can be allowed for by replacing each electrode with a number of thin strip electrodes connected to the same potential. The other effects, which are localised near the electrodes or the wall, can be modelled by discrete electrical components external to the duct. Our future research plans include improving the duct model in this way and investigating the behaviour of real consolidation circuits in the presence of non-linear electrode arc characteristics and under fault conditions.

Table 1. Current distribution for maximum output power in a consolidated group with anode 5 faulty

	1	2	3	4	5	6	7
A	1.06	1.06	1.10	1.26	0	1.27	1.13
k	0.98	0.99	0.99	0.99	0.99	0.98	0.98

Table 2. Interelectrode voltage distribution for currents of Table 1.

	1-2	2-3	3-4	4-5	5-6	6-7
A	1.06	1.08	1.18	0.11	1.15	1.20
k	0.98	0.99	0.99	0.99	0.99	0.98

Table 3. Possible current distribution in consolidated group with anode 5 faulty.

	1	2	3	4	5	6	7
A	1	1	1	1	0	1	1
k	1	1	1	1	0	1	1

Table 4. Interelectrode voltage distribution for currents of Table 3.

	1-2	2-3	3-4	4-5	5-6	6-7
A	0.97	0.96	0.93	0.08	0.92	1.07
k	1.03	1.04	1.07	0.92	0.08	0.93

References

1. R.J. Rosa, "Voltage Consolidation and Control Circuits for Multiple-Electrode MHD Generators", 15th Symposium on Engineering Aspects of Magnetohydrodynamics, Philadelphia, VII.5, 1976.

2. A. Lowenstein, "Consolidation and Local Control of Power in an MHD Generator", 17th Symposium on Engineering Aspects of Magnetohydrodynamics, Stanford, I.1.1, 1978.

3. D.T. Trung and H.K. Messerle, "Power Output of Segmented MHD Faraday Generators with Hall Voltage Compensation", Energy Conversion and Management, 21 (2), 1981.

4. H. Hurwitz, Jr., R.W. Kilb, and G.W. Sutton, "Influence of Tensor Conductivity on Current Distribution in an MHD Generator", J. Appl. Phys., 32, 205, 1961.

5. A. Solbes and A. Lowenstein, "Electrical Non-Uniformities and Their Control in Linear MHD Channels", 15th Symposium on Engineering Aspects of Magnetohydrodynamics, Philadelphia, IX.3, 1976.

6. M. Abramowitz and I.A. Stegun, "Handbook of Mathematical Functions", Dover, New York, 1972.

7. S. Kuo, B. Cheo and E. Levi, "Electrical Nonuniformities in Diagonally Connected Generators", J. Energy, 4 (2), 88, 1980.

8. R.J. Rosa, "Magnetohydrodynamic Energy Conversion", McGraw-Hill, New York, 1968.

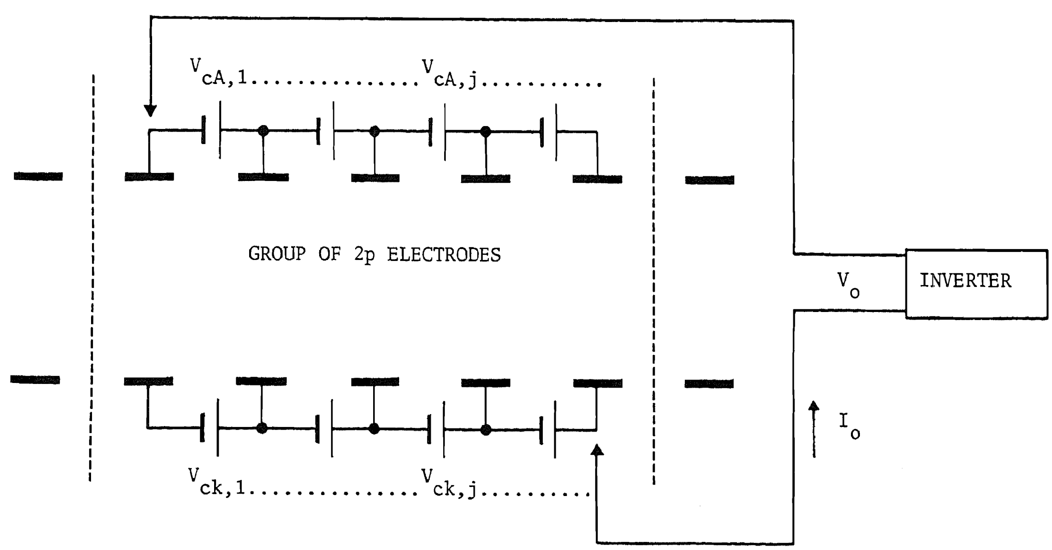

Figure 1. Idealised consolidation circuit.

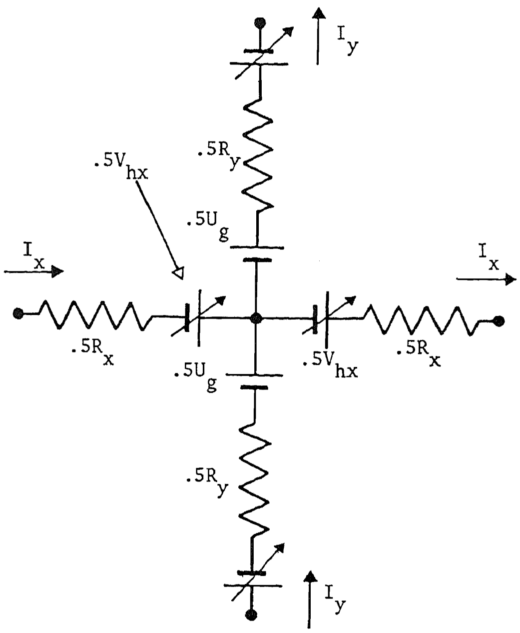

Figure 2. Equivalent circuit of a single pair of MHD generator electrodes.

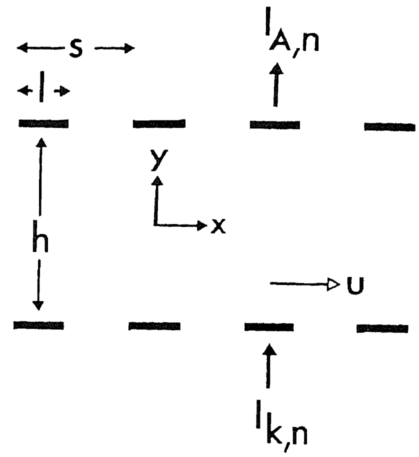

Figure 3. Model geometry.

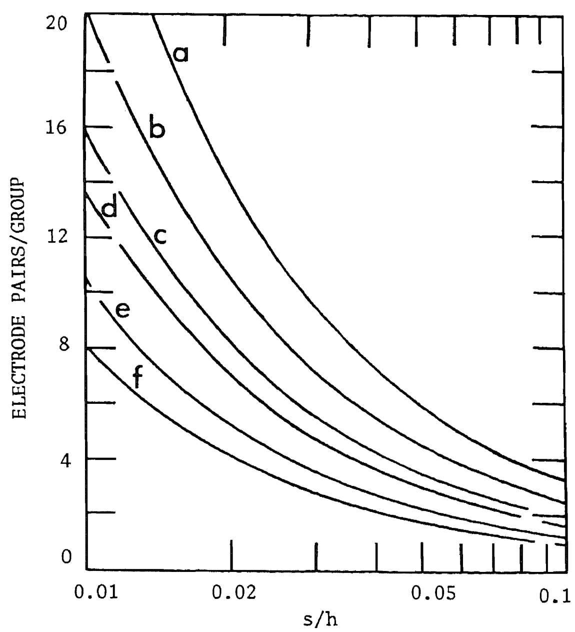

Figure 4. No. of electrode pairs which can be consolidated as a function of s/h. For curves a,b,c, F = 0.1 and η_e = 0.8, 0.75, 0.7 respectively. For curves d,e,f, F = 0.05 and η_e = 0.8, 0.75, 0.7 respectively. θ = 0.5 and β = 3 for all curves.