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A ONE-DIMENSIONAL ARC COLUMN MODEL AND ITS EFFECT ON BOUNDARY LAYER ARC BEHAVIOR*

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In a continuing effort to predict the size of the arcs on the electrodes of an MHD generator (Ref. 1), a one-dimensional model of an arc column is analyzed; this model has the advantage of being describable by an ordinary second order differential equation for which analytic solutions are readily obtained.

Consider a strip of gas of width Z flowing in the x direction (Fig.1): Z is taken to be the width of the arc column (defined as the region in which the electrical conductivity is about 1000 mho/m). The one-dimensional enthalpy equation for such a strip is

$$\rho u \frac{dh}{dx} - \frac{d}{dx}\left(K \frac{d(h/c_p)}{dx}\right) = Q \qquad (1)$$

where Q represents the net enthalpy source per unit volume. Joule heating contributes a term σE^2 to Q. It is assumed that σ = 1000 mho/m within the column and σ = 0 elsewhere; i.e. this term exists between x = 0 and x = X_0 for a column of depth X_0 positioned with its leading edge at x = 0. Conduction perpendicular to the flow is modeled by a loss term

$$\frac{-2(h - h_\infty)}{c_p Z\left(\frac{Z}{2K} + \frac{1}{S_t \rho u c_p}\right)}$$

where h_∞ is the enthalpy far from the column; the parenthetic factor in the denominator represents a thermal resistance over half the strip width in series with a film coefficient at the strip edge. It is convenient to introduce the length $\frac{1}{g} = \frac{\kappa}{\rho u c_p}$ and the nondimensional variables

$$t = \frac{h}{h_\infty} \qquad \begin{matrix}\xi = gx \\ \xi_0 = gX_0\end{matrix} \qquad \eta = gz\sqrt{1 + \frac{2}{S_t gZ}} \qquad \alpha = \frac{\sigma_a E_a{}^2 \kappa}{\rho^2 u^2 c_p h_\infty}$$

The enthalpy equation then becomes (assuming constant molecular properties)

$$\frac{d^2t}{d\xi^2} - \frac{dt}{d\xi} + \frac{4}{\eta^2}(1-t) + \alpha = 0 \qquad 0 < \xi < \xi_0$$

$$\frac{d^2t}{d\xi^2} - \frac{dt}{d\xi} + \frac{4}{\eta^2}(1-t) = 0 \qquad \xi < 0 \qquad (2)$$

The boundary conditions are: t = 1 at $\xi = -\infty$ and t = R at both ξ = 0 and $\xi = \xi_0$ where R = h_a/h_∞, h_a being the enthalpy at which σ = 1000 mho/m.

The solution of this equation is in the form

$$t = C_1 e^{\lambda_1 \xi} + C_2 e^{\lambda_2 \xi}$$

where

$$\lambda_{1,2} = \tfrac{1}{2}\left(1 \pm \sqrt{1 + (4/\eta)^2}\right)$$

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Fitting the boundary conditions yields a transendental equation for ξ_0:

$$[R-1 + \frac{\alpha\lambda_2\eta^2}{4(\lambda_1-\lambda_2)}]e^{\lambda_1\xi_0} - \frac{\alpha\lambda_1\eta^2}{4(\lambda_1-\lambda_2)} e^{\lambda_2\xi_0} + 1-R + \frac{\alpha\eta^2}{4} = 0 \quad . \tag{3}$$

The model is completed with the assumption that for a fixed current, the arc will adjust its dimensions to minimize the electric field, or what is the same thing, that for a fixed electric field the current will be an extremum. Since the arc current is

$$I = \sigma_a E_a X_0 Z, \tag{4}$$

this condition becomes (with $\xi_0 = \xi_0(\alpha,Z)$ from (3))

$$\frac{d}{dz}(Z\xi_0) = 0 \quad . \tag{5}$$

A necessary condition for the existence of a solution of (3) with $\xi_0 > 0$ is that the coefficient of $e^{\lambda_1\xi_0}$ be negative; i.e.

$$\alpha > - \frac{4(R-1)(\lambda_1 - \lambda_2)}{\lambda_2\eta^2} \quad .$$

This is equivalent to a lower bound on the electric field:

$$E_a^2 > \frac{8\Delta h\rho^2u^2c_p}{\sigma\kappa} \left[\frac{\sqrt{1 + (4/\eta)^2}}{\eta^2(\sqrt{1 + (4/\eta)^2} - 1)}\right]$$

where $(R-1)h_\infty = h_a - h_\infty = \Delta h$.

It can be shown that as E_a approaches this lower limit, subject to condition (5), η must increase so that the term in brackets approaches its minimum value of 1/8, and E can indeed approach an absolute limit

$$E_a > E_0 \equiv \rho u\sqrt{\frac{c_p\Delta h}{\sigma\kappa}} \quad . \tag{6}$$

A study of equation (3) in the opposite limit $\alpha \to \infty$, $\eta \to 0$ (but $\alpha\eta^2 = 0(1)$) yields

$$\xi_0 = \frac{-\eta}{2} \ln\left(1 - \frac{8(R-1)}{\alpha\eta^2}\right) \quad .$$

Since as $\eta \to 0$, $gZ \to \frac{1}{2}S_t\eta^2$, condition (5) becomes

$$\frac{d}{d\eta}\left[\eta^3 \ln\left(1 - \frac{8(R-1)}{\alpha\eta^2}\right)\right] = 0 \qquad \text{or}$$

$$\ln\left(1 - \frac{8(R-1)}{\alpha\eta^2}\right) = \frac{2}{3}\left(\frac{1}{1 - \frac{\alpha\eta^2}{8(R-1)}}\right) \qquad \text{or}$$

$$\frac{\alpha\eta^2}{R-1} = 14.9928; \; \xi_0 = 0.381344\eta \qquad \text{hence}$$

$$E_a = 3.327\sqrt{\rho u S_t}\left(\frac{\kappa\Delta h^3}{\sigma c_p}\right)^{\frac{1}{4}} I^{-\frac{1}{2}} \quad . \tag{7}$$

This is the result of Reference 1 (where $S_t = 0.181$; a value of this magnitude being reasonable for flow past an object as small as an arc column). It is now clear, however, that this result is only for small arcs. As the arc current increases, the column diameter increases. Eventually the resistance to heat flow inside the column becomes larger than the resistance outside, i.e. the Biot number of the column becomes larger than one. This forces the enthalpy at the center of the column to rise significantly relative to that at the edge. To provide this additional energy input, the field must rise relative to the field of a constant enthalpy column. The net result is a field that becomes constant above a certain current, which is consistent with what is commonly observed in arcs.

Figure 2 is a plot of the numerical solution of Equations (3) and (5). The dotted curve is the expression

$$E_a = \rho u\sqrt{\frac{c_p\Delta h}{\sigma\kappa}} + 3.327\sqrt{S_t\rho u}\left(\frac{\kappa\Delta h^3}{\sigma c_p}\right)^{\frac{1}{4}} I^{-\frac{1}{2}} \equiv E_o + \frac{b}{\sqrt{I}} \tag{8}$$

obtained simply by adding (6) and (7). This simple expression clearly exhibits the essential features of the exact solution (of this inexact model) and is adopted in the ensuing calculations in this paper.

The arc current and column length are determined by assuming that the total cross channel voltage is a minimum. Two minor departures from the treatment of Reference 1 have been made. A slight refinement in the treatment of the outer portion of the boundary layer is introduced by limiting the mixing length to 0.085δ and by reducing the shear and heat transfer by the factor $1-(y/\delta)^4$. Because the geometry of the "spreading region" is not well understood, two parameters were introduced to permit modeling of various geometries. It is assumed that in this region, the area of the spreading current plume can be represented by

$$A = \phi y^{2P} = \frac{2\pi}{p^{2P}}(1-\cos\theta)\left(\frac{R}{\sin\theta}\right)^{2(1-P)} y^{2P} \tag{9}$$

where y is the distance from some effective plume origin inside but very near the top of the arc column. R is the effective column radius and θ is the half angle of the plume at the top of the column. (The earlier "hemispherical" spreading is recovered with p=1, θ=π/2.) Matching this area to the area of the column and assuming as before that most of the voltage drop occurs in a distance comparable to the column dimensions, the spreading voltage becomes

$$V_s = \frac{\sigma_a\left(1-\frac{1+n}{2p}\right)E_a}{(2p-1-n)(\phi)^{\frac{1+n}{2p}}\sigma_o\tau_o{}^n u_t{}^n}\left(\frac{I}{E_a}\right)^{\frac{1+n}{2p}} \equiv cE_a\left(\frac{I}{E_a}\right)^{\frac{1+n}{2p}} \tag{10}$$

(where σ_o, τ_o and n model the 3 body recombination ionization relaxation introduced in Reference 2). The condition that the total voltage be stationary with respect to I and column height y_t becomes

$$\int_0^{y_t} b\,dy = c\left(\frac{I}{E_a}\right)^{\frac{1+n}{2p}}\left[\frac{1+n}{p}E_a\sqrt{I} - \left(1-\frac{1+n}{2p}\right)b\right]$$

$$\frac{\beta^2 j_c}{\langle\sigma\rangle+\sigma_{leak}} - \frac{(1+\beta^2)j_c}{F_c\sigma_t} + E_a + \left(\frac{I}{E_a}\right)^{\frac{1+n}{2p}}\left[E_a\frac{dc}{dy_t} + \left(1-\frac{1+n}{2p}\right)c\frac{dE_a}{dy_t}\right] = 0 \tag{11}$$

In solving these equations, the variation in all parameters except u_t, σ_t and κ through the boundary layer are neglected. The velocity comes directly from the boundary layer theory; κ (equation 8, etc.) is taken to be the effective molecular plus turbulent thermal conductivity

$$\kappa = \kappa_0 \left(1 + \frac{P_r}{P_{r_t}} \varepsilon_M\right) \qquad (12)$$

where κ_0 is the molecular conductivity at the mean boundary layer conditions. The temperature is determined by integrating

$$\frac{dT}{dy} = \frac{\dot{q}}{\kappa} = \frac{\dot{q}_0(1 - (y/\delta)^4)}{\kappa} \qquad (13)$$

to yield the electrical conductivity

$$\sigma_t = 1.78(10^6)e^{-30220/T} \quad . \qquad (14)$$

Figure 3 summarizes the results. The steep portion of the solid curves are consistent with Reference 1 and the data discussed therein. The introduction of a minimum arc E_0 somewhat slows the growth of arc current with core current density as opposed to the prediction of the simpler model. However the most striking change, namely the reduction in slope at high current, is due to the assumption of a constant mixing length in the outer portion of the boundary layer. This well illustrates the dominant role that the characteristics of the boundary layer play in determining the characteristics of the arc.

Figures 2 and 3 are calculated assuming hemispherical spreading ($\theta = 90°$ and $P = 2$). As a matter of general interest, Figure 4 indicates how the results would change for different choices of spreading geometry. The change is significant. However, the original choice together with the assumed relation for spreading conductivity seems to give good agreement with experiment, limited though the experimental data be.

If one attempts to find a solution to Equation (2) for the region behind the column, one finds that the enthalpy must drop to the ambient enthalpy, h_∞, in a distance comparable to the dimensions of the arc. This implies that the existence of a stationary arc column in a high velocity fluid stream requires the existence of a powerful mechanism for removing heat from the back side of the column. A possible mechanism is turbulent mixing between the steam of arc-heated gas and the ambient fluid stream as illustrated in Figure 5. The interface between streams having different velocity tends to become unstable at a low Reynolds number, especially if the turbulence level of the surrounding fluid is high. Experiments described in Reference 3 suggest the importance of turbulent mixing for the maintenance of a steady arc column.

REFERENCES

1. Rosa, R.J., "Boundary Layer Arc Behavior (II)," 22nd Symposium on the Engineering Aspects of MHD, Mississippi State University, June 26-28, 1984.

2. Rosa, R.J., "A Survey of Boundary Layer Arcing," 21st Symposium on the Engineering Aspects of MHD, p. 4.1.1 - 4.1.12, 1983.

3. Demetriades, S.T., C.D. Maxwell, G.S. Argyropoulos and G. Fonda-Bonardi, 11th Symposium on the Engineering Aspects of MHD, Cal. Tech., pp. 64-69, 1970.

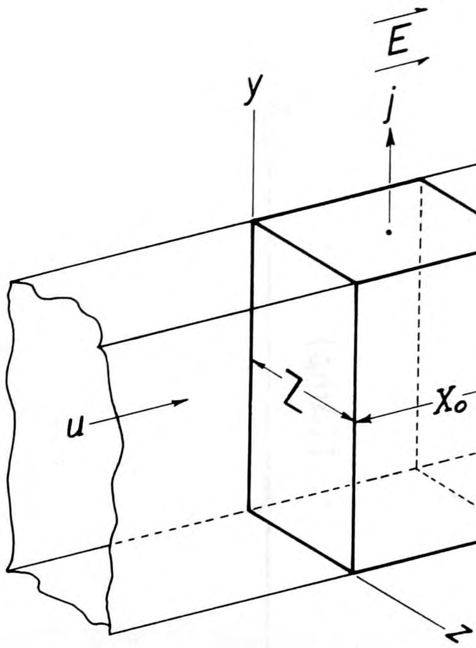

Figure 1. One-dimensional arc column model. Conductivity is 1000 mha/m for $0 < x < X$ and zero elsewhere.

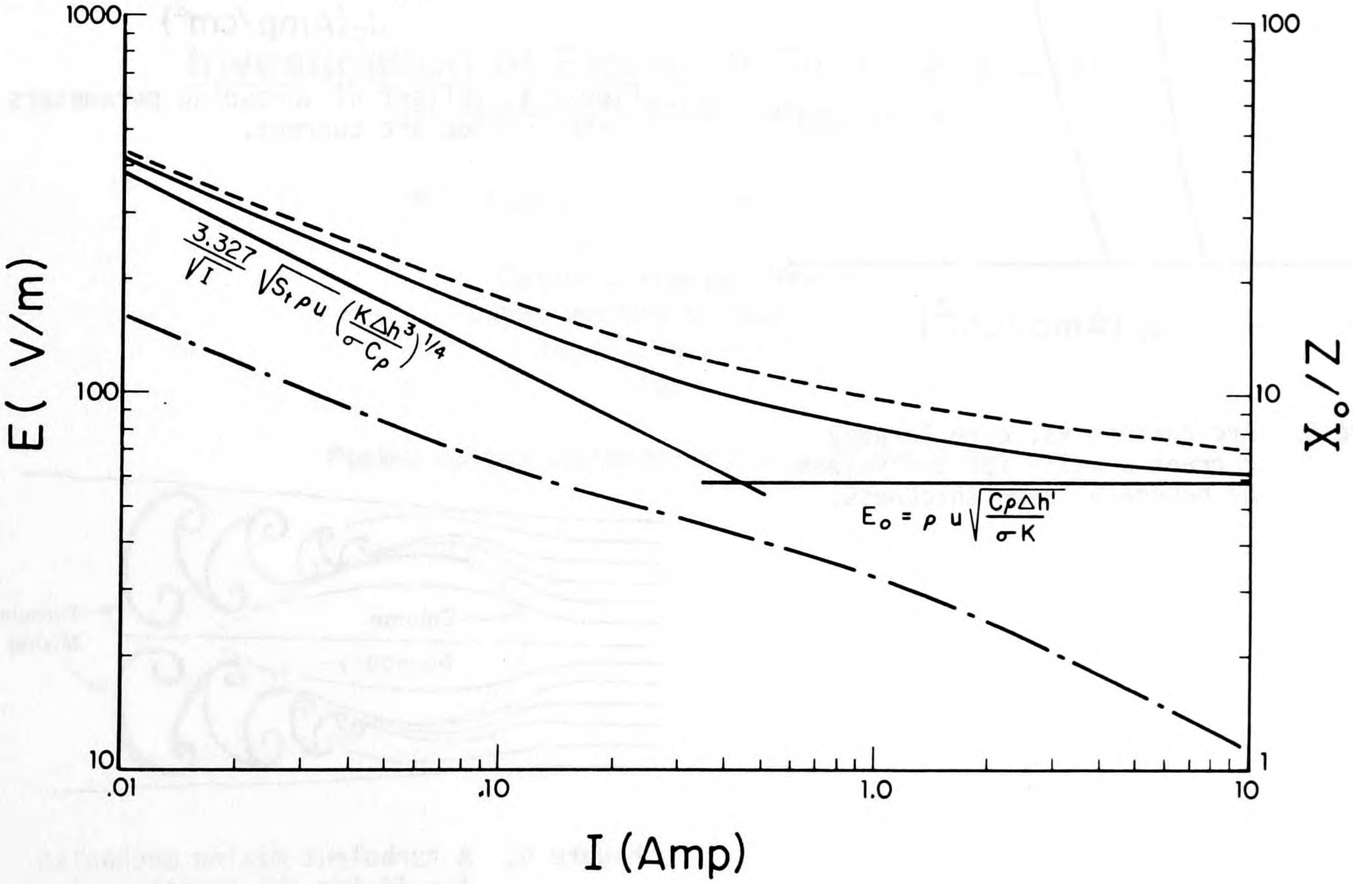

Figure 2. E-I characteristic of an arc.
Solid curve: exact solution of Equations 3 and 5.
Dashed curve: Equation 8.
Dash-Dot curve: Aspect ratio of column section.
Parameters: $\rho = 0.14$ kg/m³ $u = 414$ m/s $\sigma = 1000$ mho/m
$C_p = 1700$ j/kgK $h = 4.8(10^6)$ j/kg $k = 8$ W/mK

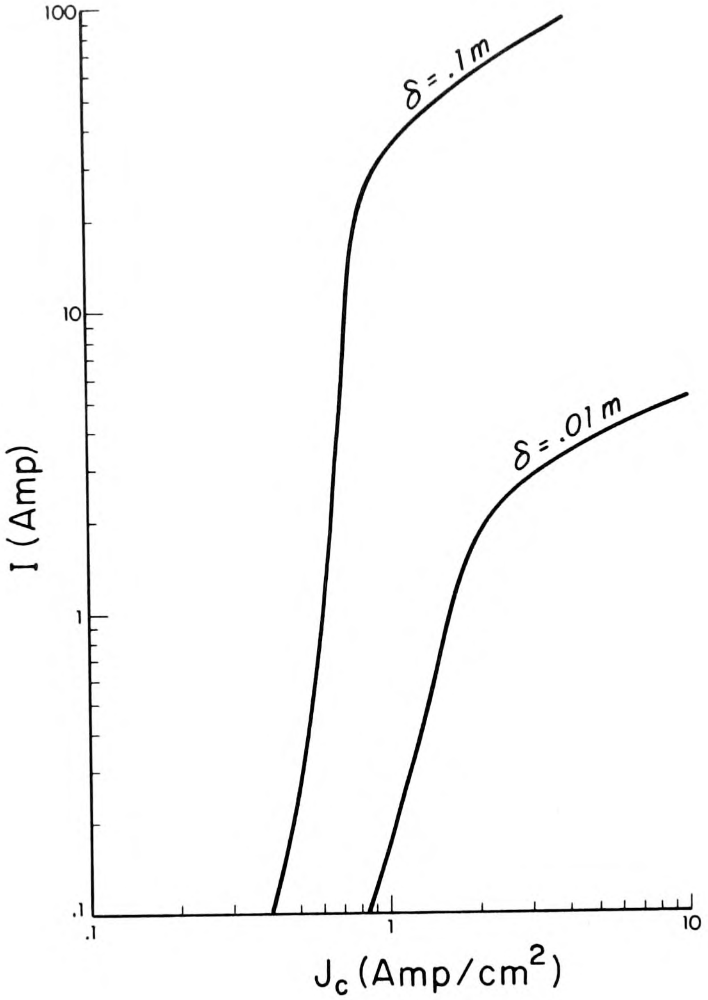

Figure 3. Arc current vs. core Faraday current density for two values of boundary layer thickness.

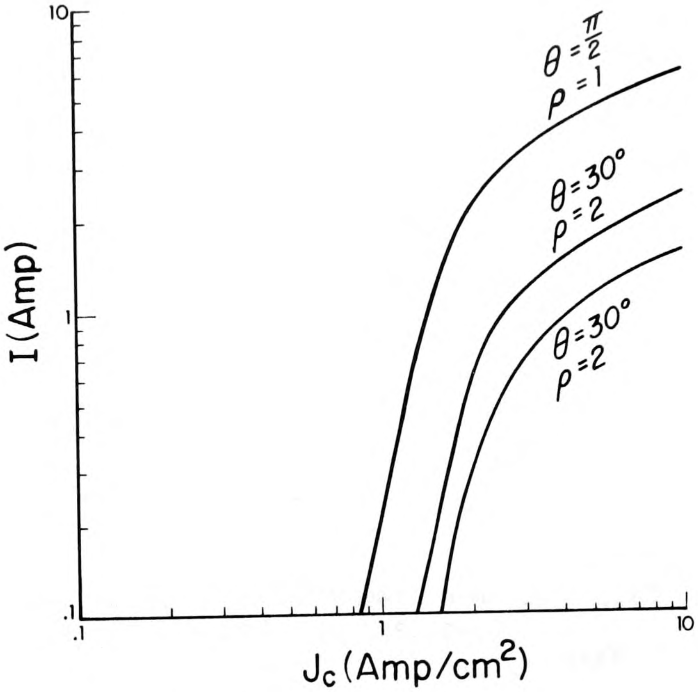

Figure 4. Effect of spreading parameters on arc current.

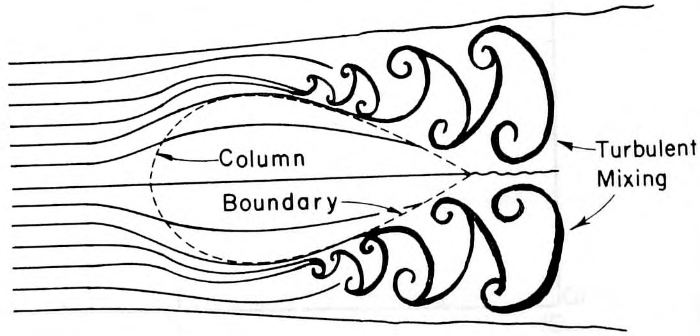

Figure 5. A turbulent mixing mechanism for fixing the position of an arc column.