Current And Potential Distributions In Finitely Segmented MHD Generators With Non-Uniform Electrical Conductivity

Author(s): D. Oliver and M. Mitchner

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CURRENT AND POTENTIAL DISTRIBUTIONS IN FINITELY SEGMENTED MHD GENERATORS WITH NON-UNIFORM ELECTRICAL CONDUCTIVITY

D. Oliver and M. Mitchner
Department of Mechanical Engineering
Stanford University
Stanford, California

Abstract

In early studies of the nature of electrical conduction in an ionized fluid in segmented electrode MHD generator structures (1-6), attention was directed to the electrodynamic aspect of the problem based on the assumption that the fluid was a medium of uniform electrical conductivity. The currents were then related to the electric field by a simple "Ohm's Law" and the problem could be treated as a classical problem in potential theory.

Recently, however, attempts to understand the results of experiments with segmented electrode MHD generator configurations have stimulated interest in the effects in the conduction process which are peculiarly due to non-uniformities in the fluid (7-9). Because of the finite size of segmented electrodes and the Hall effect, the current distribution in such structures is non-uniform and hence produces non-uniform Joulean heating. The resulting non-uniform electron temperature leads to a non-uniform electrical conductivity which in turn influences the distribution of current. Other mechanisms are present which may play a role in determining the electron temperature, conductivity, potential, and current distributions, and their importance must still be assessed. Among these are convection, conduction, and inelastic collision effects in the energy equation, diffusion of electrons due to electron pressure and temperature gradients, diffusion of ions, finite rate ionization and recombination, and sheath and emission-absorption effects at the electrode and insulator surfaces. The important inelastic collision effects appear to be ionization and atomic excitation with subsequent radiation.

For two dimensional current flow in the (x,y) plane, the current distribution $J(J_x, J_y)$ resulting from a potential distribution $\phi(x,y)$ is described by the equations:

$$\nabla \cdot J = 0 \tag{1}$$

where for negligible diffusion of electrons due to electron pressure and temperature gradients and for negligible diffusion of ions,

$$J_x = \frac{\sigma(x,y)}{1+\beta^2}\left[\beta\frac{\partial\Phi}{\partial y} - \frac{\partial\Phi}{\partial x}\right] \tag{2}$$

$$J_y = \frac{\sigma(x,y)}{1+\beta^2}\left[-\frac{\partial\Phi}{\partial y} - \beta\frac{\partial\Phi}{\partial x}\right] \tag{3}$$

and $\Phi(x,y)$ is defined by

$$\phi = \Phi - \int_0^y u(\xi)\,B\,d\xi \;. \tag{4}$$

The quantity $u(y)$ is the fluid mass velocity assumed to be known and to flow in the x-direction, and B is the applied magnetic induction assumed to be constant and to lie along the z-axis. The quantity β is the Hall parameter. The electrical conductivity $\sigma(x,y)$ is given by the simple free-path result

$$\sigma(x,y) = \frac{n_e(x,y)\,e^2}{m_e \nu_e} \tag{5}$$

where $n_e(x,y)$ is the electron number density, $-e$ the electron charge, m_e the electron mass, and ν_e the average electron collision frequency for momentum interchange. The conductivity distribution $\sigma(x,y)$ is determined from $n_e(x,y)$ and in an exact calculation, $n_e(x,y)$ is determined by solving the appropriate fluid species conservation equations.

Two cases have been considered for the study of the effects of a non-uniform conductivity. In the first, the electrical conductivity distribution is assumed, and the above equations are then solved for $\Phi(x,y)$ and $J(x,y)$. The implications of particular distributions of conductivity with regard to $\Phi(x,y)$ and $J(x,y)$ may then be examined. In the second case, the electron number density is determined by the electron temperature in the Saha equation assuming that ionization equilibrium prevails. The electron temperature is in turn determined by the electron energy equation. Hence one considers $\sigma = \sigma(n_e)$ and $n_e = n_e[T_e(x,y)]$ where, in the absence of convection, conduction, and inelastic collision effects, the energy equation for $T_e(x,y)$ is

$$\frac{3}{2}\,k\,n_e(T_e)\nu_e\left(2\frac{m_e}{m_n}\right)(T_e - T) = \frac{J^2(x,y)}{\sigma(T_e)} \tag{6}$$

where m_n is the average heavy particle mass and k is the Boltzmann constant. Proceeding in this manner, if the heavy

particle temperature, T, is assumed known, then equations (1 to 3), (5), and (6) may be solved simultaneously for $\Phi(x,y)$ and $T_e(x,y)$.

The boundary conditions for both these cases are on $\Phi(x,y)$ and are:

at an electrode
(in the x-z plane)

$$\frac{\partial \phi}{\partial x} = 0 \text{ or } \Phi = \text{constant}$$

at an insulator
(in the x-z plane)

$$J_y = 0 \text{ or } \frac{\partial \Phi}{\partial y} = -\beta \frac{\partial \Phi}{\partial x} .$$

Solutions of the equations for the cases described above have been obtained numerically. The effects of a region of either high or low conductivity near the electrodes relative to the channel center were examined using an assumed conductivity profile in which the conductivity either increased or decreased near the electrode walls. It was found that when the conductivity was high near the electrode wall, the concentration of current near the electrode edge was increased and the difference in voltage between successive electrode pairs (Hall Voltage) was decreased relative to the uniform conductivity case. When the conductivity near the electrode wall was low, the current distribution over the electrode was more uniform but the Hall voltage difference between successive electrodes was negligibily changed relative to the uniform conductivity case.

When the electron number density is determined by the Saha equation at the electron temperature, it is found that slight amounts of non-equipartition between electron and gas temperature give rise to more intense current concentrations at the electrode edge and a decreased Hall voltage between successive electrode pairs. The internal resistance of each electrode pair is found to decrease due to the enhanced conductivity; but this decrease is much less than that for infinitely fine segmented electrode structures.

References

1. H. Hurwitz, G. W. Sutton, H. Kilb, J. Appl. Phys. 32, 205, 1961.

2. F. Shultz-Grunow and D. L. Denzel, "Evaluation of the Performance Characteristics of a Faraday Generator," Engineering Developments in Energy Conversion, ASME, New York (1965).

3. L. S. Dzung, "The Magnetohydrodynamic Generator with Hall Effect at the Duct Ends," Brown Boveri Review 49, 6, (1962).

4. G. W. Sutton, "End Losses in Magnetohydrodynamic Channels with Tensor Electrical Conductivity and Segmented Electrodes," J. Appl. Phys. 34, 2, (1963).

5. F. Fishman, "Effect of Electrode Non-Uniformities Along the Magnetic Field in MHD Generators," AVCO-Everett Research Report 176 (1963).

6. J. C. Crown, "Analysis of Magnetogasdynamic Generators Having Segmented Electrodes and Anisotropic Conductivity," United Aircraft Corp. Report R-1852-2(196).

7. J. L. Kerrebrock, AVCO-Everett Research Laboratory Report 201, Everett, Massachusetts (1965).

8. J. L. Kerrebrock, AVCO-Everett Research Laboratory Report 201, Everett, Massachusetts (1965).

9. F. S. Sherman, "Non-Equilibrium Ionization and Magnetohydrodynamic Phenomenon," General Electric Space Science Laboratory Report R64SD36 (1964).