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LINEAR STABILITY OF REVERSE COMBUSTION FOR IN SITU COAL GASIFICATION

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Linear Stability of Reverse Combustion
for In Situ Coal Gasification

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ABSTRACT

A linear stability analysis of reverse combustion in coal has shown that the process is only conditionally stable. The reverse combustion linking process is unstable for the subbituminous coal properties of the Hanna No. 1 seam and for the operating conditions utilized during the Laramie Energy Research Center's in situ coal gasification field tests conducted at Hanna, Wyoming. With the aid of several simplifying assumptions, a universal neutral stability curve is obtained; that is, the locus of the dimensionless neutrally stable wavelengths (combustion finger diameters) is a unique function of a single dimensionless parameter involving heat removal from the combustion front and heat generated at the front. The curve describing the most highly amplified wavelength provides an estimate of the diameter of the combustion fingers, 2.9 feet, which is in good agreement with estimates based on thermal field test data. The analysis also provides insight into how high pressure air requirements can be scaled for the reverse combustion linking process for different well spacing.

INTRODUCTION

The linked vertical well process is one of three in situ coal gasification processes being considered in the U.S. at this time. This process employs a reverse combustion link to devolatilize a portion of the coal seam to effect a highly permeable path for the subsequent forward combustion step. The first field tests of this process in the U.S., which were conducted at the Hanna, Wyoming, field site of ERDA Laramie Energy Research Center, suggest that the reverse combustion process has a rather poor sweep efficiency, whereas the forward combustion step has a remarkably high (circa 85%) sweep efficiency. That is, the reverse combustion step appears to develop a combustion front which fingers rather than carbonizes all the coal via a smooth planar flame front. However, this fingering of the reverse combustion link appears to be beneficial to the overall combustion process because it relatively quickly provides a few highly permeable paths (fingers) for the combustion gases to escape during the subsequent forward combustion step.

The tendency for reverse combustion to finger and forward combustion to burn in a planar fashion presumably is related to the stability of these two combustion processes; that is, the tendency for reverse combustion to develop a wavelike fingered structure is somewhat analagous to the tendency of a body of water to develop waves under the action of the wind. Plain and simply, a wavelike structure is more attractive to the process from an energetic standpoint.

An understanding of the stability of the reverse and forward in situ combustion processes would be invaluable for scaling-up and improving this process. For example, the present field studies at Hanna, Wyoming, have been conducted with a well spacing of sixty feet at a reverse combustion air injection velocity averaging 70 to 100 ft^3/ft^2 hr. Under these conditions, the reverse combustion step appears to be unstable and to develop fingers having a wavelength of approximately three feet. Clearly, it is desireable to increase the well spacing for commercial scale gasification in order to minimize

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the drilling costs. However, it is not obvious how to scale-up this process for wider well spacings. If the fingering process is beneficial to the reverse combustion link, then it may be important to keep the finger size the same. Stability theory would suggest that the finger size is strongly dependent on the air injection rate. Thus, if finger size is important or perhaps can be optimized, it is of value to discern precisely how it depends on the various parameters of the process.

A stability analysis of this process may also provide useful information as to the value of laboratory bench-scale experiments for determining the propensity of these combustion processes to finger. For example, some laboratory studies indicate that the reverse combustion process is stable, whereas field tests indicate the contrary. The explanation for this may lie in the fact that the reverse combustion process may be unstable only to very long wavelength fingers which are far too large to observe in laboratory reactors.

It is clear that the confidence we can place in scaling-up from test site operations to commercial scale gasification depends on a thorough understanding of the entire in situ coal gasification process. The propensity to finger, or instability of the combustion fronts, is a poorly understood but nonetheless very significant aspect of this process which demands further study. A more complete understanding of reverse combustion appears especially important because the Soviet literature indicates that in some cases attempts to establish a reverse combustion link failed in as many as 30 per cent of the wells used. Massive failure on this scale would seriously disrupt any commercial gasification project. This paper will consider the stability of the reverse combustion step in the linked vertical well process. The stability of forward combustion will be examined in a subsequent paper.

STABILITY THEORY APPROACH

Our principal concern in this paper is the tendency for fingering of the flame front in the reverse combustion step in the in situ coal gasification process. This fingering, in turn, is related to the stability of the combustion process. Stability theory attempts to determine whether the equations describing a physical system have a tendency to depart from their original state when the dependent variables are perturbed. In a physical sense, these perturbations emanate from, say, flow rate pulsations, slight heterogeneities in the porous media, minute variations in the heating value of the coal, etc. The question that stability theory seeks to answer is whether a process, when subjected to perturbations, will return to its undisturbed state or will gravitate to a new state. Physically, the new state could be a fingered rather than a smooth planar combustion front. Stability theory attempts to determine whether a process is stable or not, and the properties of any unstable process. These properties include the propagation or growth rates of the disturbances (fingers,) and their wavelength. The manner in which these properties depend on the process variables is of paramount importance in stability studies.

In summary, then, this analysis of reverse combustion stability is intended to answer more than the simple question of whether the process is stable or unstable. Other important questions are: (1) If the reverse combustion process is unstable, can it be stabilized, and, if so, under what conditions? (2) What is the diameter of the fingers formed during an unstable reverse combustion process? (3) Can air requirements for the linking process be predicted, and how are air requirements related to coal properties and well spacing? (4) Can the stability of the reverse combustion process be studied via laboratory bench scale experiments?

RELATED STABILITY STUDIES

The importance of stability considerations in secondary and tertiary oil recovery operations by water-flooding and dilute surfactant flooding techniques has led to considerable development of stability theory for flow through porous media. Representative studies include those of Saffman and Taylor (1958), Chuoke et al. (1959), Perrine (1961a, 1961b), and Gogarty et al. (1970). These studies indicate that if one phase (say water) is displacing another immiscible phase (say oil), the displacement process will be unstable (finger) if the displacing fluid has a higher mobility than does the displaced fluid.

Unfortunately, the instabilities encountered in the in-situ coal gasification process involve more than just mobility considerations. Thermal and expansion effects due to the combustion must also be taken into account. Considerable work has been done on flame stability [see, for example, Markstein (1969)] which indicates that volume expansion effects at the flame front are in general destabilizing factors. Only recently have investigators turned their attention to the more complex stability problems associated with thermal effects and flame fronts in porous media [Armento and Miller (1976); Miller (1973, 1975); and Sherwood and Homsey (1975)].

No stability studies of the in-situ coal gasification process itself have appeared in the literature. The reason for this is readily apparent. In order to do a stability analysis, one first needs to have solved the appropriate differential equations describing the unperturbed or stable process. The latter has been done only recently for in-situ coal gasification [Magnani and Farouq-Ali (1975); Edgar and Dinsmoor (1976); Gunn and Whitman (1976); Thorsness and Rozsa (1976); and Kotowski and Gunn (1976)]. Thus, it is possible only now to proceed with a stability analysis of this process.

SOLUTION FOR UNPERTURBED COMBUSTION

Since stability theory seeks to determine whether perturbations on the unperturbed or quiescent solution to this combustion process will grow or decay, we must first develop the latter solution. The model to be developed here is a somewhat simplified version of the solution of Kotowski and Gunn (1976).

Consider the idealization of the reverse combustion process shown in Figure 1. Air flows in the $+\eta^*$ direction where η^* is a coordinate measured from the moving combustion front. The carbonized coal is region II and the unburned coal is region I. Heat conduction from the combustion front vaporizes all the water at the steam front located at $-\eta^*_{sf}$.

It is convenient to cast the conservation equations in dimensionless form using the following dimensionless variables:

$$\overline{T} = \frac{\overline{T}^* - T_v}{T_f - T_v} \; ; \; \overline{v} = \frac{\overline{v}^*}{\overline{v}^*_g} \; ; \; \eta = \frac{\eta^*}{\eta_s} \qquad (1)$$

where

$$\eta_s \equiv \frac{k(T_v - T_f)}{\rho_a \omega_o \Delta H_R \overline{v}^*_g} \qquad (2)$$

The length scale factor η_s is a measure of the penetration depth of conduction from the combustion front in region I. Its definition is suggested by balancing the heat generation term with the conduction term in the energy balance at the combustion front. The gas injection velocity $\overline{v}^*_g$ and combustion front velocity $\overline{v}^*$ are assumed constant in this analysis. The remaining parameters appearing above and elsewhere in this paper are defined in the nomenclature section. The corresponding dimensionless form of the energy equation is then given by

$$\frac{d^2\overline{T}}{d\eta^2} - N_c \frac{d\overline{T}}{d\eta} = 0 \qquad (3)$$

where

$$N_c \equiv N_R[1+(N_p+\epsilon_I)\overline{v} \simeq N_R(1+N_p\overline{v}) \qquad (4)$$

$$N_R \equiv \frac{\eta_s \rho_g \overline{v}^*_g C_g}{k} = \frac{C_g(T_v - T_f)}{\omega_o \Delta H_R} \qquad (5)$$

$$N_p \equiv \frac{\rho_s C_s}{\rho_g C_g} \qquad (6)$$

The dimensionless group N_R provides a measure of the ratio of the capacity to remove heat from the combustion front to the rate at which heat is generated. The group N_p is a ratio of the thermal capacity of coal to that of the gas.

The boundary conditions are given by

$$\overline{T} = 1 \text{ at } \eta = 0 \qquad (7)$$

$$\overline{T} = 0 \text{ at } \eta = -\infty \quad (8)$$

The latter boundary condition, specifying that $\overline{T}^* = T_v$ at the steam front, can be applied at $\eta^* = -\infty$ if the following condition is satisfied:

$$-\frac{\rho_a \omega_o \Delta H_R \overline{v}_g^*}{\rho_w \Delta H_v \overline{v}^* \phi_I S_w} >> 1 \quad (9)$$

If the above condition is satisfied, the behavior of the combustion front can be decoupled from the influence of the steam front, which considerably simplifies the analysis.

In order to solve eq. 3 subject to the boundary conditions given by eqs. 7 and 8, it is necessary to know the interrelationship between T_f, $\overline{v}^*$, and $\overline{v}_g^*$, since for a specified $\overline{v}_g$ and given physical properties they are not independent. One relationship is provided by the energy balance at the combustion front given in dimensionless form by

$$\frac{d\overline{T}}{d\eta} - 1 = 0 \text{ at } \eta = 0 \quad (10)$$

A second relationship is obtained by assuming the front temperature to be a linear function of the gas injection velocity:

$$\overline{T}^* = A^* + B^* \overline{v}_g^* \quad (11)$$

The numerical solution of Kotowski and Gunn (1976) suggests that $A^* = 695°F$ and $B^* = 2.88°F \cdot hr/ft$ for Hanna No. 1 subbituminous coal. This assumption obviates the need to solve the convective diffusion equation with the associated reaction kinetics. Including the latter would unnecessarily complicate the analysis.

The solution to eq. 3 satisfying eqs. 7 and 8 is given by

$$\overline{T} = e^{N_c \eta} \quad (12)$$

Equation 10 requires that

$$N_c = 1 \quad (13)$$

The latter provides a solution for the front velocity as a function of the injection gas velocity and the parameters of the process, which agrees well with the more exact analysis of Kotowski and Gunn (1976). The solution for the unperturbed temperature profile can now be used in obtaining a solution for the perturbed combustion process.

SOLUTION FOR PERTURBED COMBUSTION

This analysis follows the formalism of linear stability theory wherein infinitesimal perturbations in all the dependent variables are assumed to be of the form

$$T = \overline{T} + T' \; ; \; v_g = \overline{v}_g + v_g' \; ;$$

$$v = \overline{v} + v' \; ; \; P = \overline{P} + P' \quad (14)$$

The resulting linearized form of the dimensionless energy equation is given by

$$\frac{\partial^2 T'}{\partial \eta^2} + \frac{\partial^2 T'}{\partial y^2} + \frac{\partial^2 T'}{\partial z^2} - N_R e^{\eta} - \frac{\partial T'}{\partial \eta} = N_p N_R \frac{\partial T'}{\partial t} \quad (15)$$

The corresponding boundary conditions are given by

$$T' \to 0 \text{ as } \eta \to -\infty \quad (16)$$

$$T' + \eta' = B v_g' \text{ at } \eta = 0 \quad (17)$$

where

$$B \equiv B^* \overline{v}_g^* / (T_f - T_v) = 1 + \frac{C_g (A^* - T_v)}{\omega_o \Delta H_R N_R} \quad (18)$$

The temperature and velocity perturbations are related via the perturbed form of the energy balance at the combustion front:

$$\frac{\partial T'}{\partial \eta} + \eta' - v_g' - \phi_I v' = 0 \text{ at } \eta = 0 \quad (19)$$

In arriving at eq. 19 we have assumed that heat removal from the combustion front due to the perturbed temperature in region II can be ignored relative to heat removal in region I. This assumption is plausible if heat removal is principally by conduction; however, it may not be reasonable if radiative heat losses are significant. Ignoring the heat losses in region II obviates the need for solving the perturbed energy equation in this region.

The velocity perturbations must satisfy Darcy's equation and the continuity equation:

$$\vec{v}_g' = -\nabla P' \quad (20)$$

and

$$\nabla \cdot \vec{v}_g' = 0 \quad (21)$$

where

$$P' \equiv P'^* k_I / \mu_g \eta_s \quad (22)$$

The perturbed velocity must satisfy the boundary condition

$$\vec{v}_g' \rightarrow 0 \text{ as } \eta \rightarrow -\infty \quad (23)$$

For very large mobility ratios $N_k \equiv k_{II}\mu_g/k_I\mu_c$, characteristic of the reverse combustion process in coal, the continuity of pressure condition at the combustion front implies that

$$\eta' = P' \quad (24)$$

where η' is the dimensionless perturbation of the combustion front.

The functional forms for T', v_g', P' and η' which satisfy eqns. 17, 19, 20, 21, and 24 are given by

$$T' = \tau(\eta) f(y,z) e^{\beta t} \quad (25)$$

$$v_g' = \alpha a_g f(y,z) e^{\alpha\eta} e^{\beta t} \quad (26)$$

$$P' = -a_g f(y,z) e^{\alpha\eta} e^{\beta t} \quad (27)$$

$$\eta' = -a_g f(y,z) e^{\beta t} \quad (28)$$

where a_g is the amplitude of the perturbation of the combustion front and $f(y,z)$ satisfies the equation

$$\frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = -\alpha^2 f \quad (29)$$

This equation is satisfied by simple harmonic functions (waves or fingers) having wave number $\alpha = 2\pi\eta_s/\lambda = (\alpha_y^2 + \alpha_z^2)^{\frac{1}{2}}$, where α_y and α_z are the wave numbers in the y- and z-directions respectively. For two dimensional disturbances corresponding to fingers in the form of slits, either $\alpha_y = 0$ (fingered slits parallel in the z-plane) or $\alpha_z = 0$ (fingered slits parallel in the y-plane). However, for the homogeneous coal seam assumed here one would anticipate three-dimensional disturbances with $\alpha_y = \alpha_z$; hence, $\alpha = \sqrt{2}\alpha_y$, or $\lambda_y = \lambda_z = 2\lambda$. That is, the fingers will have a circular cross-section with diameter 2λ. In reality, the permeability of coal perpendicular to the bedding plane cannot be the same as the permeability parallel to the bedding plane; however, little information is available concerning such directional permeability in a subbituminous coal. In the absence of more definite information, homogeneity is assumed.

If eqs. 25 through 28 are substituted into eq. 19, an ordinary differential equation is obtained for $\tau(\eta)$, the amplitude of the temperature perturbation. The solution to this equation which satisfies eqs. 16, 17, and 19 is given by

$$\{\alpha + \frac{B}{(1 - N_R)}\alpha(\alpha - \gamma) - \frac{\gamma}{(1 - N_R)}\}$$
$$\{\frac{1}{2} + \frac{1}{2}[1 + 4(\gamma + \alpha^2)]^{\frac{1}{2}}\} - \alpha(1 + \alpha)$$
$$+ \frac{\gamma(1 + \alpha)}{(1 - N_R)} = 0 \quad (30)$$

where,

$$\gamma \equiv (N_p + \phi_I)N_R\beta \simeq N_p N_R \beta \quad (31)$$

In arriving at the above we have assumed that $\phi_I << N_p$. Equation 30 constitutes a relationship between the parameters α, the wave number of the disturbance or finger, and γ, the dimensionless growth coefficient of the disturbance.

In principle, eq. 30 can be solved for γ as a function of the parameters of the problem which include α, B, and N_R. Note that eqs. 25 through 28 indicate that positive values of γ imply unstable perturbations of wave number α which grow in amplitude with time. Negative values of γ imply stable perturbations. The locus of wave numbers satisfying the condition $\gamma = 0$ defines the neutral stability curve corresponding to disturbances which neither grow nor decay. For neutrally stable disturbances eq. 30 reduces to

$$[1 + \frac{B}{(1 - N_R)}\alpha_n][\frac{1}{2} + \frac{1}{2}(1 + 4\alpha_n^2)^{\frac{1}{2}}] - 1 - \alpha_n = 0 \qquad (32)$$

Equation 32 implies that the neutrally stable wave number α_n is a unique function of the dimensionless group $B/(1 - N_R)$. For all values of $B/(1 - N_R)$, $\alpha_n = 0$ is a root of eq. 32; that is, $\alpha_n = 0$ is one branch of the neutral stability curve. The neutral stability curve has a bifurcation point at $B/(1-N_R) = 1$ which determines an uppermost gas velocity above which this process is stable to all disturbances of nonzero wave number. Equation 32 has been solved for the nonzero neutrally stable wave number as a function of the group $B/(1-N_R)$ using a Newton-Raphson numerical method.

This analysis indicates that there will be one disturbance with a maximum γ corresponding to the most highly amplified wave. This is the perturbation which should be observed in practice since it grows more rapidly than all the others. The most highly amplified wave number and its growth coefficient correspond to solutions of eq. 30 satisfying the conditions

$$\frac{\delta\gamma}{\delta\alpha} = 0; \qquad \frac{\delta^2\gamma}{\delta\alpha^2} < 0 \qquad (33)$$

The roots α_m and γ_m satisfying eqs. 30 and 33 were found numerically using a multivariable Newton-Raphson technique. Equation 30 implies that α_m is a function only of B and N_R. However, eq. 18 indicates that for fixed gas and coal properties, α_m will be a function only of N_R.

DISCUSSION OF RESULTS

Before discussing the linear stability analysis, it is of interest to consider the implications of the simplified unperturbed combustion model developed here. Equation 13 implies that the dimensionless unperturbed combustion front velocity is given by

$$\bar{v} = (\frac{1}{N_R} - 1)/(N_p + \phi_I) \qquad (34)$$

Reverse combustion corresponds to $\bar{v} > 0$, whereas forward combustion implies that $\bar{v} < 0$. Hence, the transition between reverse and forward combustion will occur at a gas velocity satisfying the condition

$$N_R = 1 \qquad (35)$$

In terms of dimensional quantities this implies that reverse combustion will revert to forward combustion for gas velocities exceeding

$$\bar{v}_g^* > (-\frac{\omega_o \Delta H_R}{C_g} - A^* + T_v)/B^* \qquad (36)$$

For the typical properties of Hanna No. 1 subbituminous coal given in Table I, eq. 36 implies that forward combustion will occur when $\bar{v}_g^* > 1100\ ft^3/ft^2 \cdot hr$. To maintain the reverse combustion link the gas velocities should not exceed this value; however, since the quantitative accuracy of eq. 36 has not been verified, this limiting gas velocity can be considered only an estimate.

The upper solid line and the line defined by $\alpha_n = 0$ in Figure 2 constitute the neutral stability curve for the physical properties and operating conditions indicated in Table I.

Table I. Physical properties and operating conditions for Hanna field tests

ρ_s	= 90 lb/ft^3
C_s	= 0.67 Btu/lb °F
C_g	= 0.30 Btu/lb °F
K	= 0.25 Btu/hr·ft °F
ΔH_R	= - 5500 Btu/lb
ϕ_I	= 0.15
P	= 3 atm
ω_o	= 0.2 lb O_2/lb gas

Note that the neutral stability curve is a unique function of the dimensionless group $B/(1-N_R)$ as indicated by eq. 32. The bifurcation point in the neutral stability curve occurs at $B/(1-N_R) = 1$. For the properties indicated in Table I this corresponds to a gas velocity of 285 ft^3/ft^2·hr. That is, this analysis implies that this in-situ reverse combustion process will be stable with respect to all nonzero wave number disturbances for velocities exceeding 285 ft^3/ft^2·hr. As the group $B/(1-N_R)$ (or gas velocity) decreases below unity (or 285 ft^3/ft^2·hr) there is a band of unstable wave numbers which extends from $\alpha = 0$ to an upper limit which progressively increases. This band of unstable wave numbers assymptotically approaches infinite width as $B/(1-N_R) \to 0$. This asymptote is only of theoretical interest since it corresponds to a zero gas velocity. In practice it is difficult to sustain reverse combustion at very low air injection velocities.

This then implies that at all gas velocities in the range $0 \leq \overline{v}_g^* \leq 285$ ft^3/ft^2·hr the reverse combustion link will be unstable in that there are some wave numbers which will be amplified. According to this analysis it is possible to stabilize this process by using very large gas injection velocities.

The following physical explanation emerges to describe the nature of the instability of the in-situ reverse combustion process. The energy source for any unstable disturbance is, of course, the combustion reaction occurring at the front. Any process which tends to remove energy from the combustion front has a stabilizing influence on the process since it deprives the disturbances of this energy. The mechanism by which unstable disturbances are propagated in this process is illustrated in Figure 3. When the unperturbed combustion front is subjected to a disturbance leading to a wavy perturbed front, a point such as "A" in Figure 3 experiences an increase in gas velocity due to the higher mobility region penetrating into the lower mobility region. This increase in gas velocity increases the combustion at point "A" and causes the front to finger further at this point. Similar but converse arguments show that points such as "B" will be retarded, thus further enhancing the fingering. The only stabilizing influence included in this analysis is heat conduction from the combustion front in region I. Heat conduction preferentially stabilizes the large wave numbers or short waves since short waves have a larger surface area for a given amplitude. As the gas velocity [or $B/(1-N_R)$] increases, heat conduction progressively stabilizes longer and longer waves until it completely stabilizes the combustion process when the group $B/(1-N_R) = 1$. This progressive stabilization with increasing gas velocity arises because the parameter η_s, which is a measure of the penetration depth of the conduction, decreases as $\overline{v}_g^*$ increases. Smaller values of η_s imply steeper temperature gradients and correspondingly greater conduction from the front.

It is of interest to speculate on the implications of the assumptions embodied in this analysis. The principal assumptions in this analysis are that the gases are incompressible; the permeability ratio N_K is infinite; that heat conduction from the combustion front in the devolatilized region can be neglected; and, that the steam front is infinitely far removed from the combustion front. All of these neglected effects would appear to be stabilizing in that they remove energy from the combustion process, and hence deprive the disturbances of energy needed for their growth. Inclusion of any of these neglected stabilizing effects would lower the neutral stability curves. The stabilizing factors neglected in this analysis will be considered in a forthcoming paper [Gunn and Krantz (1977)].

The lower solid line in Figure 2 corresponds to the locus of the most highly amplified wave number or most unstable finger. Equation 18 and 30 imply that for a fixed set of coal and gas properties, α_m will be a function only of N_R, or equivalently, $B/(1-N_R)$, since B is uniquely determined by N_R for these conditions. The dashed line in Figure 2 is the locus of the most highly amplified wave number for long waves (c_m=0); its equation is given by

$$\alpha_m = 0.463[1 - B/(1 - N_R)] \quad (37)$$

This asymptotic limit for α_m is useful in predicting the finger size under actual in-situ reverse combustion coal gasification where the gas injection flux continuously decreases during the initial stages of injection as the well bore increases in diameter.

In principle, Figure 2 permits one to estimate the finger size, since one would anticipate that this would correspond to the wavelength of the most unstable disturbance. However, this determination is difficult in practice since the gas injection flux is not known. An estimate, however, can be obtained in the following way. Initial gas injection velocities are approximately 3000 ft^3/hr. The initial flux then will exceed 285 ft^3/ft^2hr. Figure 2 would then indicate that the reverse combustion process is initially stable. As the combustion zone around the well bore progressively widens, the air flux decreases. Hence, let us determine the wavelength λ of the most highly amplified wave given by eq. 37 whose wavelength corresponds to a flux given by $1500/(\pi\lambda^2/4)$. Equation 37 gives the wave number or wavelength of a two-dimensional disturbance. The fingers observed in the reverse combustion process would appear to be circular in cross-section and hence correspond to three-dimensional disturbances. In a homogeneous medium the wavelength of the most highly amplified three-dimensional disturbance will be equal to $\sqrt{2}$ times the wave length of the most highly amplified two-dimensional disturbance determined by eq. 37 or the more general correlation given in Figure 2. For a given gas injection velocity, this wavelength is unique and given by 2.9 ft. This agrees well with estimates of the finger size for the reverse combustion link.

The Hanna II Phase II test has been the only one with sufficient instrumentation to allow an estimation of reverse combustion tube size from thermal data. These data indicated the formation of two linkage paths with a diameter of $2^1/2$ to $3^1/2$ feet (Hommert, 1977; Hommert and Beard, 1977).

CONCLUSIONS

The analysis in this paper represents an initial investigation of instability in reverse combustion. The analysis at best is expected to be semi-quantitative because of the simplifying assumptions adopted. Nevertheless, such quantities as finger size are predicted with surprising accuracy. A more accurate and much more complicated stability analysis is currently under development to determine if this excellent agreement with field test data is largely fortuitous.

In a field design of the linked vertical process, it is important to estimate the air requirements for reverse combustion linking because specially designed high pressure compressors are often used for this purpose. If finger size can be reliably predicted, the cross-sectional area for flow and correspondingly the air flux can be determined. Predictability of finger size, therefore, appears to be necessary if the linking compressors are to be properly designed.

Because the injection air appears to travel down well developed combustion fingers, reverse combustion is essentially a one-dimensional coal bed. The total air injection rate should be the same regardless of well spacing; thus the linking time will increase linearly with well spacing. These conclusions are apparently in agreement with Soviet field testing experience.

Reverse combustion during in situ coal gasification is unstable at the conditions used during the Hanna field tests. In principle, reverse combustion can be stabilized at very high air flux rates; however, the Hanna coal is not very permeable (3 to 10 millidarcies) and the needed flow rates may not be attainable for reasonable well spacings. Lignite and lower grade subbituminous coal are much more permeable, and reverse combustion stabilization at high air flux rates may be possible.

In summary the major conclusions of this work are: 1. Reverse combustion is unstable at the operating conditions used during the Hanna field tests. Very high air flux can stabilize the process. Any mathematical model, however, which ignores combustion instability cannot correctly predict reverse combustion linking results for field tests conducted at normal operating conditions. 2. Calculated combustion front finger sizes agree well with field test data. 3. The air injection rate should be held constant irrespective of well spacing. Linking time should increase linearly with distance. 4. If air flow rates are properly scaled, combustion instabilities can be reproduced in laboratory size equipment.

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NOMENCLATURE

- A — dimensionless constant in eq. 11 = $A^*/(T_f-T_v)$
- a_g — dimensionless amplitude of perturbed combustion front
- B — dimensionless constant in eq. 11 = $B^*\bar{v}_g/(T_f-T_v)$
- C_g — heat capacity of gas in region I
- C_s — heat capacity of solid in region I
- f — function defined by eq. 29
- ΔH_R — heat of combustion
- ΔH_v — heat of vaporization of water
- k_I — permeability in region I
- k_{II} — permeability in region II
- N_c — dimensionless constant defined by eq. 4
- N_k — $= k_{II}\mu_g/k_I\mu_c$
- N_p — dimensionless constant defined by eq. 6
- N_R — dimensionless constant defined by eq. 5
- P — pressure
- S_w — relative saturation of water
- t — time
- T — temperature
- T_f — temperature of combustion front
- T_v — temperature of steam front

v	velocity of combustion front	
v_g	velocity of gas in region I	
x	stationary coordinate measured from injection well	
y	lateral coordinate	
z	vertical coordinate	

Greek Letters

α	dimensionless wave number $= 2\pi n_s/\lambda$
α_m	most highly amplified wave number
α_n	neutrally stable wave number
α_y	dimensionless wave number in y-direction
α_z	dimensionless wave number in z direction
β	dimensionless frequency $= \beta^* n_s/\bar{v}_g$
γ	dimensionless frequency $= (N_p + \phi_I)N_R\beta$
γ_m	maximum amplification rate
η	coordinate measured positive from combustion front into region II
η_s	scale factor defined by eq. 2
η_{sf}	η-coordinate of steam front
λ	wavelength
μ_c	shear viscosity of gas in region II
μ_g	shear viscosity of gas in region I
ρ_a	density of injected air
ρ_g	density of gas in region I
ρ_s	density of coal
ρ_w	density of liquid water
τ	dimensionless amplitude of temperature perturbation
ϕ_I	porosity in region I
ω_o	weight of oxygen per unit weight of gas in region I

Superscripts

$\bar{}$	denotes time-average or unperturbed quantity
*	denotes dimensional quantity
'	denotes perturbed quantity

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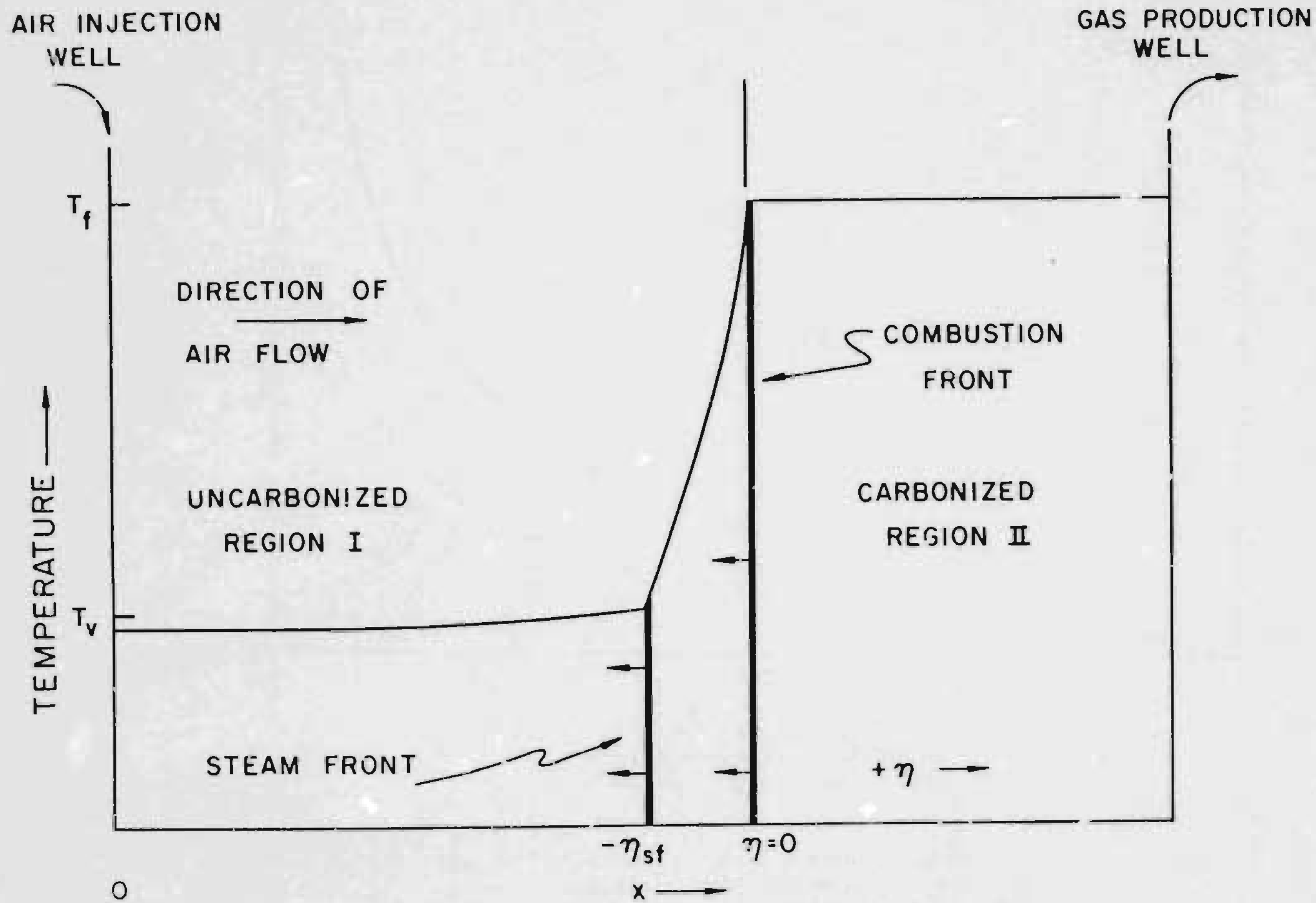

Figure 1. Idealization of Reverse Combustion Process

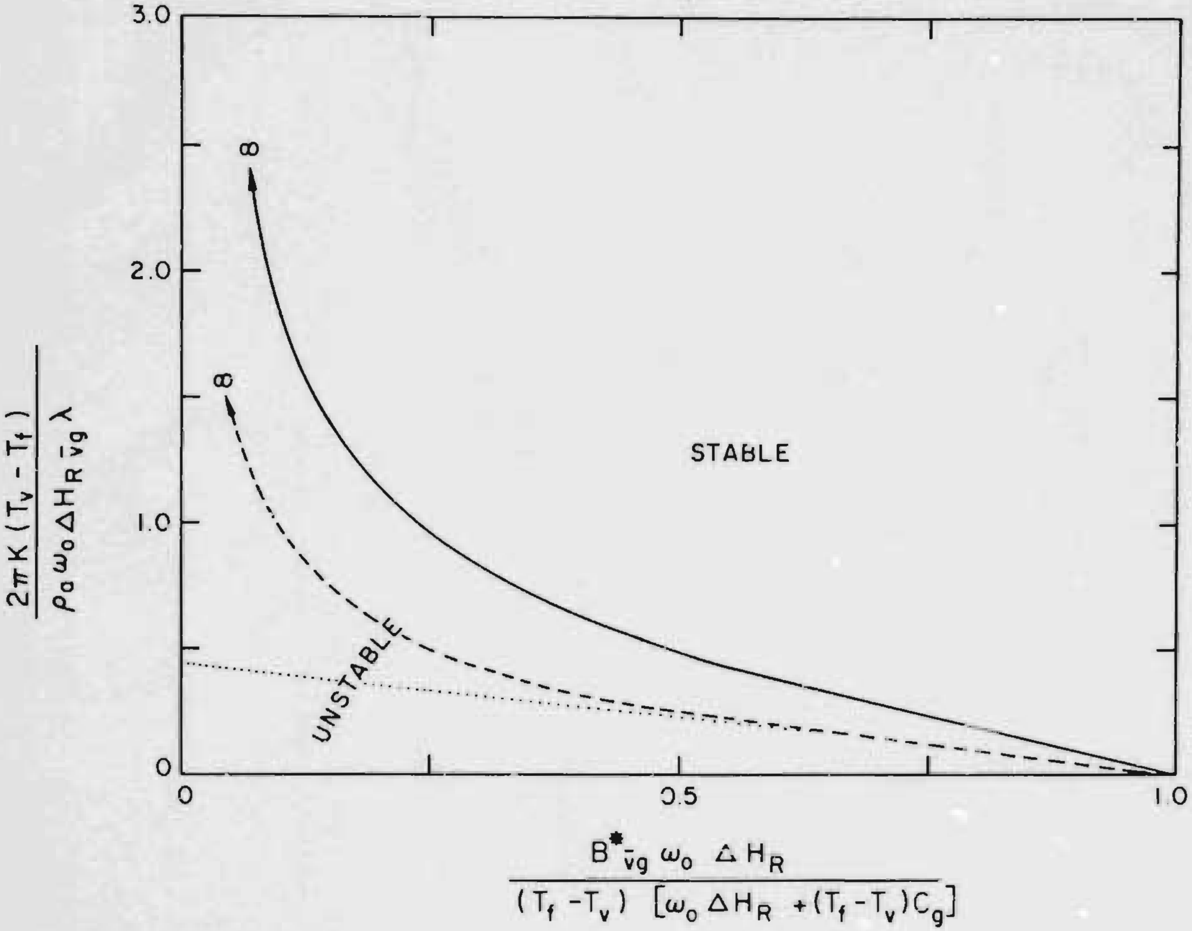

Figure 2. Neutrally Stable and Most Highly Amplified Wave Numbers as a Function of the Dimensionless Group $B/(1-N_R)$

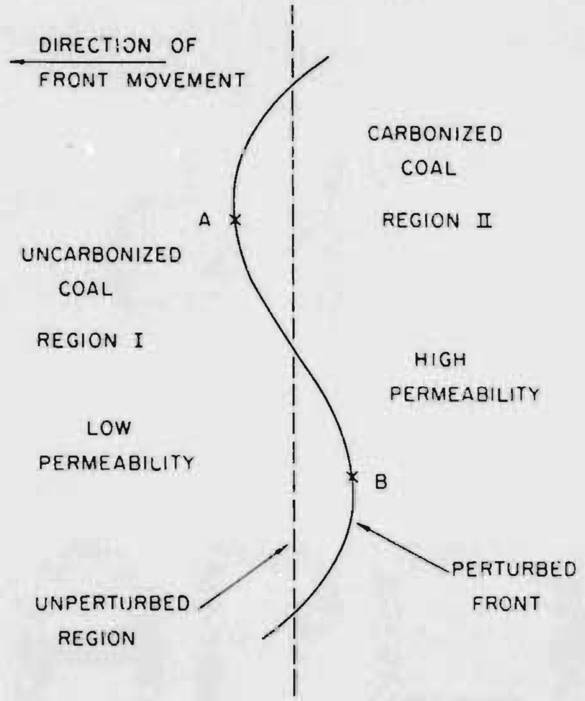

Figure 3. Mechanism by which Unstable Disturbances are Propagated in Reverse Combustion

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